Swingtum – A Computational Theory of Fractal Dynamic Swings and Physical Cycles of Stock Market in A Quantum Price-Time Space

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(Paper for the 2003 Hawaii International Conference on Statistics and Related Fields)

Abstract
This paper presents the basic framework of a comprehensive computational theory of stock market behavior, which we call Swingtum, taking multivariate stock index time series data as input, and producing probabilistic predictions of stock index movement at multiple time frames. The theory should also be applicable to other liquid markets. The Swingtum theory is based on the view that the movement of the stock market as a whole as represented by its benchmark index is driven by three types of forces: business dynamics, mass psychological dynamics, and news impacts, and consequently the market movement can be decomposed into four types of components: dynamical swings, physical cycles, abrupt momentums, and random walks. Dynamic swings include business cycles, stock life cycles, and Elliott waves of different levels, which typically have a fractal nature characterized by log-periodic power laws. Physical cycles includes anniversary days, seasonality cycles, and weekly cycles, which have relatively constant periodicity. Abrupt momentums may be caused by endogenous critical points or driven by exogenous news shocks or impacts. Random walks correspond to remaining randomness not explainable by any systematic force. The dynamic swings and physical cycles identified and modeled from the historical index time series will most likely define a quantum space of price and time in which the market will most likely travel from one quantum price level to another or from one time zone to another. There is a fundamental symmetry between price and time. The actual path is not only determined by dynamic swing phase and physical cycle phase, but also by the possible news impact. The more general theory of Swingtum extends the fractal and cyclical models of a univariate benchmark index to the multivariate time series models of intramarket and intermarket dynamical analysis.

Keywords: Computational finance, stock market index, dynamic swing, physical cycle, abrupt momentum, random walk, fractal, log-periodic power law, time series prediction, Elliott waves, Gann cycles, Gann angles, price-time symmetry, quantum space, news impact.

Acknowledgement: An early stage of this research was sponsored by China's National Natural Science Foundation under the grant "Learning Bayesian networks for knowledge discovery and data mining" through the School of Remote Sensing and Information Engineering, Wuhan University, Wuhan, China.

1. Introduction

We observe that professionals and academics from each of technical analysis, quantitative analysis and fundamental analysis of the financial markets are beginning to recognize the importance of each other's approach, and there is a tendency for the three to converge. The aim of this research is to combine technical analysis and quantitative analysis into a unified theory, which is the first major step in this convergence. Another motivation comes from our real-data neural network experiments which indicate a significant performance of over 90% correctness in predicting the next-day index change direction. It turns out both necessary and feasible to develop a comprehensive computational theory of stock market behavior, which could predict the market direction at multiple time frames using only index or price time series data. The ideas presented in this paper constitute a basic framework of such a theory which we shall call the Swingtum theory in the following and future discussions.

Stock markets are complex dynamical systems whose elements are investors and traders with varying capital sizes using different investment and trading strategies. The behavior of the stock market of a given country is generally measured by a benchmark index such as ASX S&P 200 index for Australian stock market. The index is recorded in a time series whose time unit is usually one day or one minute. The daily index time series is the primary data source for technical analysis and quantitative analysis for short-term traders and mid- to long-term investors with time frames ranging from days through years to decades. The minutely index time series is the main data source for day traders with a time frame ranging from a few minutes through hours to a few days, normally less than a week. Each data point contains usually 5 numbers: the open, high, low, close price and the traded volume of that time unit. In general, if we only consider one number, it is the close price. Still the time series of an index is considered a univariate time series, even the single variable may be a vector of 5 elements. Looking inwards, a stock market has its internal structure consisting of a number of sectors, so the complete collection of all the sector indexes for a stock market may be called the intramarket time series which must be multivariate. Looking outwards, a stock market is positioned in the international stock market ecological systems. It may have its super markets that influence itself in an asymmetrical way, and there are also a number of other interrelated stock markets on the same level, so the influences are not uni-directional. Taking the Australian stock market as an example, the US stock market as represented by three indexes – Dow Jones, S&P 500, and NASDAQ may be considered its super market, while the Japanese, Hong Kong, Korean, German, French and British stock markets are obviously its neighbors on the same level. The super and neighbor stock markets for a given stock market as well as typical bond markets, gold and oil markets and other interrelated financial markets may form the intermarkets of this stock market. The univariate time series of the stock market and the multivariate time series of its intramarkets and intermarkets provide the complete primary data source for prediction of the stock market movement. The secondary data source includes mainly the news (including all kinds of textual information) available from the Internet. However, this secondary data source is extremely irregular, and requires sophisticated techniques for automatic processing and interpretation.

This research aims at developing a comprehensive computational theory about the dynamics of the stock market for predicting the stock market index movement on multiple time frames from the primary data source of the index time series of this market, its intramarkets and its intermarkets, as well as from the automatic analysis of the news from Internet. We shall call the complete collection of our views to the stock market, our assumptions, our mathematical models as well as computational procedures the Swingtum theory. Although this theory will keep evolving, our basic view to the market is fundamental. Our inspirations come from a century-old technical analysis and the recent progresses in quantitative analysis of the financial markets, including econometrics and econophysics. A companion paper (Pan 2003) provides a joint review on technical and quantitative analysis and also has pointed out the possibility of a unified science of intelligent finance. More references on quantitative finance are provided by Sornette (2003), Farmer (1998), Farmer and Joshi (2002), Lo and McKinlay (1999), Campbell et al (1997), Mandelbrot (1982), Zhou and Sornette (2003). Comprehensive coverage on technical analysis can be found in Murphy (1999), Achelis (2000), Bulkowski (2002), and Prechter (2002).

The paper is organized as follows. Section 2 establishes our fundamental view to the stock market which is expressed in the Swing Market Hypothesis. Section 3 formulates a simple but general dynamic model of the market returns. Sections 4 and 5 presents the parametric models for characterizing fractal and cyclical market movements. Section 6 describes the quantum price-time space in which the market is supposed to travel as driven by endogenous dynamics and exogenous news shocks or impacts. Section 7 outlines a computational approach of nonparametric nearest neighbor pattern recognition exploiting multidimensional chaos in price time series. Finally section 8 concludes the paper.

2. The Swing Market Hypothesis

A long-standing conventional view of the mainstream economists to the financial markets is expressed in an Efficient Market Hypothesis (EMH), which views asset prices and their associated returns from the perspective of the speculator, and assumes the market is almost always efficient, meaning that the prices already reflected all current information that could help anticipating future events. Under the EMH, the stochastic process of market returns is modeled as uncorrelated random walk, so any profitable prediction of the market returns would be considered impossible. Although the strong form of EMH has been shown not true by many studies as reviewed by Pan (2003), EMH does provide a perfect reference with which more realistic market views can be developed.

We consider that the market is driven by various forces of different origins, wave lengths and magnitudes, as well as different durations. Each force only has one dimension: either positive for upwards or negative for downwards. At any given time, all the effective forces combine to form a joint force which has a joint market impact leading to changes in price (or index). For generality, we will just talk about the dynamics of the price of a market. If the market is a stock, the price of the market is just the price of that stock. If the market is the whole stock market for a national economy, the price of the market refers to a benchmark index such as ASX S&P 200 index for the Australian stock market.

Following Mandelbrot's discovery of fractals in the financial markets, Peters (1994) proposed a Fractal Market Hypothesis (FMH) which basically says that the market is stable when it consists of investor covering a large number of investment horizons, and information is valued according to the investment horizon of the investor. Because the different investment horizons value information differently, the diffusion of information will also be uneven. At any one time, prices may not reflect all available information, but only the information important to that investment horizon (or time frame in our terms). The FHM owes much to the Coherent Market Hypothesis (CMH) of Vaga (1991) and the K-Z model of Larrain (1991).

Like the FMH and CMH, our view to the market, which we call the Swing Market Hypothesis (SMH), is also based on the premise that the market assumes different states and can shift between stable and unstable regimes. However, we take one step further in considering the dynamic structure of the market. The Swing Market Hypothesis (SMH) proposes the following:

(1) The market always consists of investors or traders with all possible different capitals, different time frames, different information conditions, and different skills. The ever-lasting and ever-evolving differences in investors or traders are the permanent drivers of market dynamics.
(2) The market is sometimes efficient and other times inefficient, and the market tends to swing between these two modes intermittently. Each mode, efficient or inefficient, may comprise multiple regimes such as trending, cycling, spiking, consolidation, etc.
(3) The market movement is driven mainly by three types of forces: business dynamics, mass psychological dynamics, and new impacts.
(4) The market movement can be decomposed into four types of components: dynamical swings, physical cycles, abrupt momentums and random walks.

Business dynamics include global and national business cycles, intramarket dynamics and intermarket dynamics, which usually refer to the fundamental economic and business conditions. Mass psychological dynamics include the greed and fear dynamics of investors and traders defined by human nature and also forged by existing popular knowledge, methodologies and technologies of technical analysis and fundamental analysis. For example, the appearance of a certain chart pattern may trigger similar trading decisions made by many different technical traders because they have all acquired similar technical analysis education. Both business dynamics and mass psychological dynamics can produce similar dynamical swings which can be characterized by mathematical fractal models such as log-periodic power laws. Elliott waves of different levels are the visual and qualitative description of dynamical swings by technical analysts. Although not as strong as dynamical swings, physical cycles do exist in the market, which includes anniversary days, seasonality cycles and weekly cycles. For example, statistical studies show that Thursday of a week is often the reversal if the first three days have trended in the same direction. There are even intraday dynamical patterns given the daily market situation. Physical cycles have relatively constant periodicity. Abrupt momentums refer to drastic price movement which cannot be expressed in continuous analytical forms. Momentums may be caused by exogenous forces such as news shocks or impacts, or as critical points or singularities caused by endogenous dynamics. Random walks in the context of SMH correspond to remaining randomness not explicable by any systematic force.

The next section derives a simple but fundamental price impact model of the stock market returns driven by market forces.

3. A Fundamental Price Impact Model of The Stock Market

Assume the market operates in the continuous time t, i.e. all trading occurs in a continuous flow, so we shall not need to distinguish between each individual order. We will only consider the price (or index) of the market $p(t)$ as a continuous function of time. At any time t, the joint market force $f(t)$ acts upon the market and drives the price $p(t)$ up or down in a continuous flux of movement. Note that the market force $f(t)$ may include all kinds of supply or demand, greed or fear, rational decisions or emotional reactions, etc. But it must realize its impact to the market price through all kinds of orders and transactions. We shall not concern ourselves with all the order details unlike the agent-based models such as one proposed by Farmer (1998). However, we shall follow some part of Farmer's path in model building but our development is rather going into a direction quite different from his agent-based model.

Let $p(t)$ be the price (or index) at time t, the relative return $R_\tau(t)$ is defined as

$$R_\tau(t) = \frac{p(t+\tau) - P(t)}{p(t)} \tag{1}$$

where τ is the time scale. In general, it is more common to use the log-return $r_\tau(t)$ defined as

$$r_\tau(t) = \ln p(t+\tau) - \ln p(t) \tag{2}$$

For small changes in $p(t)$, the log-return $r_\tau(t)$ and the relative return $R_\tau(t)$ are approximately equal.

Suppose at time t the price $p(t)$ is changed to $\widetilde{p}(t)$ immediately after the force $f(t)$ acts upon the market, and this process can be expressed as

$$\widetilde{p}(t) = \widetilde{p}(p(t), f(t), m) \tag{3}$$

where m is the abstract mass of the market, which reflects how much the price will change relative to the magnitude of the force, and which may depend on the capitalization of the market and the past trading history.

The price change should satisfy the following basic conditions

1) The price is always positive but finite

$$0 < \widetilde{p}(p, f, m) < \infty \tag{4}$$

2) $\tilde{p}$ is an increasing function of the force f, meaning that the price impact is in the direction of the joint force and (maybe nonlinearly) proportional to force magnitude

$$\tilde{p}(p, f, m) \propto f \tag{5}$$

3) If there is no force at all there is no market impact, i.e.

$$\tilde{p}(p, 0, m) = p \tag{6}$$

4) $\tilde{p}$ is additive in the force

$$\tilde{p}(\tilde{p}(p, f_1, m), f_2, m) = \tilde{p}(p, f_1 + f_2, m) \tag{7}$$

5) $\tilde{p}$ is a decreasing function of the mass m, meaning that the price impact is inversely (maybe nonlinearly) proportional to the mass

$$\tilde{p}(p, f, m) \propto \frac{1}{m} \tag{8}$$

6) The price return due to the impact of the force f is completely determined by f and m

$$\frac{\tilde{p}}{p} = \delta(f, m) \tag{9}$$

δ is called the market impact function, and it must be an increasing function of f and a decreasing function of m according to equations (5) and (8).

Applying equation (9) into equation (7) gives a fundamental equation of the market impact function

$$\delta(f_1 + f_2, m) = \delta(f_1, m)\delta(f_2, m) \tag{10}$$

This equation has a simple but remarkable solution satisfying equations (5) and (8)

$$\delta(f, m) = e^{f/m} \tag{11}$$

Therefore, the basic price impact function is derived as

$$\tilde{p} = pe^{f/m} \tag{12}$$

The basic log-return dynamics function is thus

$$r = \ln\frac{\tilde{p}}{p} = \frac{f}{m} \tag{13}$$

Or we should remember the time,

$$r(t) = \ln\frac{\tilde{p}(t)}{p(t)} = \frac{f(t)}{m(t)} \tag{14}$$

We see that this model is similar to Newton's Second Law in physics. However, The abstract mass m is not a constant, but a slowly varying quantity in time relative to the variation of force $f(t)$. It can be understood as a scale factor that normalizes the order size and be considered as the liquidity. However, we prefer to consider it as an abstract quantity whose variation may be related to the clustered volatility. Considering the high autocorrelation of the clustered volatility, m may be considered constant for a limited time period. However, we must be aware that m may vary slowly. A varying mass may be compared to the Einstein's relativity theory in physics. Although professional traders are aware of the relativity of price in mass psychology, a modeling of price relativity requires further research.

The log-price at the current time t can be obtained through the integral of the log-return from the starting time $t = 0$ to the current time t

$$\ln p(t) = \ln p(0) + \int_0^t \ln r(t)dt \;\; = \ln p(0) + \int_0^t \frac{f(t)}{m} dt \tag{15}$$

We mainly consider two types of forces: dynamic swings and physical cycles. For generality, assume there are L levels of dynamic swing forces f_l, $l = 1,2,\cdots,L$, and K levels of physical cycle forces g_k, $k = 1,2,\cdots,K$. The joint force $f(t)$ is the sum of multilevel dynamic swing forces and multilevel physical cycle forces

$$f(t) = \sum_{l=1}^{L} f_l(t) + \sum_{k=1}^{K} g_k(t) \tag{16}$$

Substituting equation (16) into equation (15) gives

$$\ln p(t) = \ln p(0) + \int_{t_0}^t \frac{f(t)}{m} dt = \ln p(0) + \sum_{l=1}^{L} \ln p_l(t) + \sum_{k=1}^{K} \ln q_k(t) \tag{17}$$

where

$$\ln p_l(t) = \frac{1}{m} \int_0^t f_l(t)dt \qquad \text{for } l = 1,2,\cdots,L \tag{18}$$

$$\ln q_k(t) = \frac{1}{m} \int_0^t g_k(t)dt \qquad \text{for } k = 1,2,\cdots,K \tag{19}$$

are respectively the contribution to the current log-price $\ln p(t)$ by the evolutionary impact of the dynamic swing force $f_l(t)$ and the physical cycle force $g_k(t)$.

Let

$$A_0 = \ln p(0) \tag{20}$$

$$\Phi(t) = \sum_{l=1}^{L} \ln p_l(t) \tag{21}$$

$$\Psi(t) = \sum_{k=1}^{K} \ln q_k(t) \tag{22}$$

equation (17) can be rewritten as

$$\ln p(t) = A_0 + \Phi(t) + \Psi(t) \tag{23}$$

Apparently, $\Phi(t)$, $\Psi(t)$ are the joint dynamic swing component and the joint physical cycle component in the log-price.

4. Multilevel Fractal Swings In Log-Periodic Power Laws

Each l-th level swing force $f_l(t)$ must have a certain wave form. A number of studies have shown the existence of the log-periodicity in the stock market indexes and prices, thus we consider that $f_l(t)$ should include a wave form in log-time

$$f_l(t) \sim \sin(\omega_l \ln(t - \tau_l) + \phi_l) \tag{24}$$

where ω_l, ϕ_l are the log-period and phase respectively, and τ_l a time shift. Considering the existence of power laws and log-periodicity as demonstrated by Sornette's group and other researchers, we consider $p_l(t)$ should have a form which includes a power component and a log-periodic wave component

$$\ln p_l(t) \approx A_l + B_l(t-\tau_l)^{\beta_l}\ [1 + C_l \cos(\omega_l \ln(t-\tau_l) + \phi_l)] \tag{25}$$

where $A_l, B_l, C_l, \beta_l, \omega_l, \phi_l$ are unknown parameters pertinent to level l. Sornette et al have also shown the possibility of using more complicated second-order and third-order Landau expansions, however, we consider our use of multilevel log-periodicities could render higher-order Landau expansions unnecessary. Using equation (10), the joint influence of dynamic swings can be expressed as

$$\Phi(t) \approx A + \sum_{l=1}^{L} B_l(t-\tau_l)^{\beta_l}[1 + C_l \cos(\omega_l \ln(t-\tau_l) + \phi_l)] \tag{26}$$

There are two different ways for actual computation using this model. The first is a general regression of all the unknown parameters of this model using the whole time series data set. The second way is a recursive procedure which successively fits a single level model of (25) to one segment of the time series data corresponding to an Elliott wave in the context of the previous level. However, we recognize that a single log-periodic power law model of (25) can only fit to a single wave, which is what was done by Sornette's group. Such a single model cannot predict the regime shift after an anti-bubble – a correction wave for a bullish trend – has finished. In a truly fractal procedure, on each level, we should fit an elementary fractal made up of two waves: a trending wave followed by a correcting wave. This elementary fractal for an up trending wave and its correcting wave is defined by a pair of log-periodic power laws

$$\begin{cases} \ln p_l(t) \approx A_l + B_l(\tau_l - t)^{\beta_l}[1 + +C_l(\tau_l - t)^{\beta_l} \cos(\omega_l \ln(\tau_l - t) + \phi_l)] \quad \text{for } t < \tau_l & (27) \\ \ln p_l(t) \approx A'_l + B'_l(t-\tau_l)^{\beta'_l}[1 + C'_l(t-\tau_l)^{\beta'_l} \cos(\omega'_l \ln(t-\tau_l) + \phi'_l)] \quad \text{for } t > \tau_l & (28) \end{cases}$$

In this case, τ_l should correspond to the critical point – the time of reversal from the trending wave to the correcting wave. Equations (27) and (28) should be fitted to the up trending wave and the correcting wave respectively. However, we consider there may exist certain geometrical relationships between the two sets of parameters: $(A_l, B_l, C_l, \beta_l, \omega_l, \phi_l)$ versus $(A'_l, B'_l, C'_l, \beta'_l, \omega'_l, \phi'_l)$. This geometry, if existent such as Fibonacci ratios, would substantially reduce the number of unknown parameters. Observations from technical analysis show that this geometry is probabilistic.

5. Multilevel Physical Cycles in Hilbert Transform

Unlike the dynamic swings characterized by log-periodicity, physical cycles have linear periodicity and linear phase. Therefore, we can use the well-established signal processing techniques for detecting physical cycles including the periodicity and phase shift. Hilbert transform $\hat{f}(t)$ of a function $f(t)$ is defined for all t by

$$\hat{f}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{f(\tau)}{t-\tau} d\tau \tag{29}$$

Hilbert transform $\hat{f}(t)$ is orthogonal to the original function $f(t)$, so it can be used to create an analytical signal from a real signal

$$z(t) = f(t) + i\hat{f}(t) = A(t)e^{i\varphi(t)} \qquad (i = \sqrt{-1}) \tag{30}$$

where

$$A(t) = \sqrt{f^2(t) + \hat{f}^2(t)}\,, \qquad \varphi(t) = \arctan\left(\frac{\hat{f}(t)}{f(t)}\right) \tag{31}$$

are the instantaneous amplitude and the instantaneous phase. Therefore, Hilbert transform provides a basic means for modeling physical cycles of linear periodicity and phase shift of the market price time series (taken as real signals). Ehlers (2001) developed a set of practical algorithms for calculating the dominant period and instantaneous phase from price time series data. However, his approach is limited to a single level. Our model of physical cycles generalizes to multilevels. In essence, we first generalize a simple Hilbert transform to a Hilbert wavelet, and then the time series data are analyzed through a multi-scale wavelet transform into a wavelet pyramid. On each level of this pyramid, we detect the dominant periodicity and calculate the instantaneous phase from the Hilbert transform on that level. The outcome of this multilevel Hilbert transform and detection procedure will be a complete description of multilevel periodicities and instantaneous phases. Pan (1996) conducted a comparative performance study for a number of complex wavelets.

6. A Quantum Space of Price and Time

Elliott wave theory suggests that the price, more often than not, traces back to certain Fibonacci ratios such as 38.2%, 50%, 61.8%, 100%, etc. The support or resistance levels have a quantum nature (or at least geometrical). Gann was probably the first in discovering the quantum structure of the price-time space. Gann theory of price-time cycles (Pan 2003) suggests that there is a significant symmetry between the price and the time, and time is often more important than price. When certain time zones and price levels coincide, the reversal is imminent. On the other hands, the log-periodic power law models show clear geometrical properties that are consistent with the Gann angles developed in technical analysis. The dynamic swings and physical cycles identified and modeled from the historical index time series will most likely define a quantum space of price and time. This space is divided by important price levels and time zones defined by the previous dynamic swings and physical cycles. The market will most likely travel from one significant price level to another or from one significant time zone to another. The actual path is not only determined by dynamic swing phase and physical cycle phase, but also by the possible news impact.

7. Multidimensional Embedding and Nearest Neighbour Algorithm for Prediction

Based on the fractal model of dynamic swings and the wavelet analysis of physical cycles in a quantum space of price and time, we will investigate two different approaches for predicting the stock index movement The first is a direct application of the fractal model and wavelet analysis whose parameters are estimated from the historical data, especially the last and current Elliott waves. The second approach is a pattern recognition approach based on chaos theory. Each time sample from historical time series data is embedded in a multidimensional feature space, where the feature vector consists of a subset of the fractal and wavelet parameters, especially the phases at the multiple time scales. For any given current time, its feature vector is constructed from the fractal and wavelet models, and then a certain number of its nearest neighbors are searched out from the historical pattern space. Finally, a certain nonlinear regression model can be estimated from the nearest neighbors. This model then can be used for prediction. Note that this regression is local to the nearest neighbors, and thus the predictive model is adaptive.

8. Concluding Remarks

The Swingtum theory outlined in this paper provides a comprehensive dynamic model of stock market integrating fractal dynamic swings and physical cycles as well as the quantum price-time space. The model is computable in terms of statistical parameter estimation and nonparametric multidimensional embedding and nearest neighbor pattern recognition. The theory is a step toward unifying professional technical analysis and academic quantitative analysis into a science of intelligent finance. The more general Swingtum theory should extend the fractal and cyclical models of a univariate benchmark index to the multivariate time series models of intramarket and intermarket dynamic analysis.

This is an ongoing effort, further theoretical development, system implementation and real-data experiment will be reported in the future.

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